Section 7.2 – Right Triangle Trigonometry

These are the angles we will use most often in this class (as well as in your calculus classes in the future). You need to commit these angles to memory as soon as possible!

<table>
<thead>
<tr>
<th>Degrees</th>
<th>0°</th>
<th>30°</th>
<th>45°</th>
<th>60°</th>
<th>90°</th>
<th>120°</th>
<th>135°</th>
<th>150°</th>
<th>180°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radians</td>
<td>0</td>
<td>$\frac{\pi}{6}$</td>
<td>$\frac{\pi}{4}$</td>
<td>$\frac{\pi}{3}$</td>
<td>$\frac{\pi}{2}$</td>
<td>$\frac{2\pi}{3}$</td>
<td>$\frac{3\pi}{4}$</td>
<td>$\frac{5\pi}{6}$</td>
<td>$\pi$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Degrees</th>
<th>210°</th>
<th>225°</th>
<th>240°</th>
<th>270°</th>
<th>300°</th>
<th>315°</th>
<th>330°</th>
<th>360°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radians</td>
<td>$\frac{7\pi}{6}$</td>
<td>$\frac{5\pi}{4}$</td>
<td>$\frac{4\pi}{3}$</td>
<td>$\frac{3\pi}{2}$</td>
<td>$\frac{5\pi}{3}$</td>
<td>$\frac{7\pi}{4}$</td>
<td>$\frac{11\pi}{6}$</td>
<td>$2\pi$</td>
</tr>
</tbody>
</table>

Here is a circle showing all of these angles on it (plus a few more). The first number is the degree measurement; the second number is the radian measurement. Pay particular attention to the RADIAN measures of the STARRED angles.

If you learn well from playing games/quizzes, these websites quiz you on locating angles on a circle:

http://www.purposegames.com/game/angles-of-the-unit-circle-radians-quiz (for the radians quiz)

http://www.purposegames.com/game/angles-of-the-unit-circle-degrees-quiz (for the degrees quiz)
THE TRIGONOMETRIC FUNCTIONS

An **acute angle** is any angle that is between 0° and 90° (or 0 and $\frac{\pi}{2}$ radians). Drawing a vertical line between the terminal side and the initial side of the angle creates a **right triangle**. The side labeled “c” is the **hypotenuse** of the right triangle. (The hypotenuse is the side directly across from the right angle.) The leg of the triangle that is directly opposite from the angle $\theta$ is called the **opposite** side. And the leg of the triangle that is attached to the angle $\theta$ is called the **adjacent** side.

Assuming $\theta$ is an acute angle ($\theta$ can never be the right angle!), we define the six trigonometric functions based on the ratios between any two sides of this triangle. The six trig functions are Sine (sin), Cosine (cos), Tangent (tan), Cosecant (csc), Secant (sec), and Cotangent (cot), and their definitions are given in the table below:

<table>
<thead>
<tr>
<th>Function</th>
<th>Ratio</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sin \theta$</td>
<td>$\frac{\text{opposite}}{\text{hypotenuse}}$</td>
<td>$\frac{b}{c}$</td>
</tr>
<tr>
<td>$\cos \theta$</td>
<td>$\frac{\text{adjacent}}{\text{hypotenuse}}$</td>
<td>$\frac{a}{c}$</td>
</tr>
<tr>
<td>$\tan \theta$</td>
<td>$\frac{\text{adjacent}}{\text{opposite}}$</td>
<td>$\frac{a}{b}$</td>
</tr>
<tr>
<td>$\csc \theta$</td>
<td>$\frac{\text{hypotenuse}}{\text{opposite}}$</td>
<td>$\frac{c}{b}$</td>
</tr>
<tr>
<td>$\sec \theta$</td>
<td>$\frac{\text{hypotenuse}}{\text{adjacent}}$</td>
<td>$\frac{c}{a}$</td>
</tr>
<tr>
<td>$\cot \theta$</td>
<td>$\frac{\text{adjacent}}{\text{opposite}}$</td>
<td>$\frac{a}{b}$</td>
</tr>
</tbody>
</table>

You have probably noticed that $\csc \theta$ is the reciprocal of $\sin \theta$, and $\sec \theta$ is the reciprocal of $\cos \theta$, and $\cot \theta$ is the reciprocal of $\tan \theta$. These are the **Reciprocal Identities**. The easiest way to learn the trig values of certain angles is to learn only sine, cosine, and tangent of the angles, and then remember that cosecant, secant, and cotangent are merely the reciprocals of those values. **Note that the “s” of sin matches with the “c” of csc and the “c” of cos matches with the “s” of sec**. So, for instance, if you are ever trying to remember which one (sec or csc) is the reciprocal of cosine, remember that a “c” is always paired with an “s”, so it must be sec that is the reciprocal of cosine.

The **Quotient Identities** state:

\[
\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \text{and} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}
\]

You can see evidence of this using information from the table above:

\[
\sin \theta = \frac{b}{c} \quad \text{and} \quad \cos \theta = \frac{a}{c} \quad \text{So if} \quad \tan \theta = \frac{\sin \theta}{\cos \theta} \quad \text{then} \quad \tan \theta = \frac{b}{a}
\]

**Pythagorean Identities** (these come from using the Pythagorean Theorem: $a^2 + b^2 = c^2$):

\[
\sin^2 \theta + \cos^2 \theta = 1 \quad \tan^2 \theta + 1 = \sec^2 \theta \quad \cot^2 \theta + 1 = \csc^2 \theta
\]

The first and second ones are the most important and will be used often!
Example:

\[ a) \, \sin^2\left(\frac{\pi}{6}\right) + \cos^2\left(\frac{\pi}{6}\right) \]  
\[ b) \, \sec^228^\circ - \tan^228^\circ \]

Here's something to help you remember what sine, cosine, and tangent are: "Soh Cah Toa"!

"SOH" means: \( \text{Sine} = \frac{\text{Opposite}}{\text{Hypotenuse}} \)

"CAH" means: \( \text{Cosine} = \frac{\text{Adjacent}}{\text{Hypotenuse}} \)

"TOA" means: \( \text{Tangent} = \frac{\text{Opposite}}{\text{Adjacent}} \)

Then, remember:

- **Cosecant is the reciprocal of Sine.**
- **Secant is the reciprocal of Cosine.**
- **Cotangent is the reciprocal of Tangent.**

Important Algebra Review on Rationalizing Denominators

Recall from your Intermediate and/or College Algebra class that we should never leave a radical in the denominator of a fraction. The act of getting rid of the radical is called "Rationalizing the Denominator". To **rationalize a denominator** when it contains a single radical, just multiply the numerator and denominator by the exact radical that is in the denominator.

Example: Rationalize the Denominator in each of the fractions.

\[ a) \, \frac{2}{\sqrt{5}} \quad b) \, \frac{1}{\sqrt{2}} \quad c) \, \frac{3}{\sqrt{3}} \]
**Finding the Values of the Trigonometric Functions When One Is Known**

Given the value of one trigonometric function of an acute angle \( \theta \), the exact value of each of the remaining five trigonometric functions of \( \theta \) can be found in either of two ways.

**Method 1 Using the Definition**

**STEP 1:** Draw a right triangle showing the acute angle \( \theta \).

**STEP 2:** Two of the sides can then be assigned values based on the value of the given trigonometric function.

**STEP 3:** Find the length of the third side by using the Pythagorean Theorem.

**STEP 4:** Use the definitions in equation (1) to find the value of each of the remaining trigonometric functions.

**Method 2 Using Identities**

Use appropriately selected identities to find the value of each of the remaining trigonometric functions.

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**Example:** Find the value of the six trigonometric functions of the angle \( \theta \) in each figure.

![Diagram](image.png)
Example: Find the exact value of each of the remaining five trigonometric functions of the acute angle $\theta$.

a) $\csc \theta = 5$

b) $\sec \theta = \frac{5}{3}$

COMPLEMENTARY ANGLE THEOREM

The trig functions can be grouped into three pairs of cofunctions. The cofunction pairs are 1) sine and cosine, 2) secant and cosecant, and 3) tangent and cotangent. (Notice that the second function of each cofunction pair is just the first function with the "co-" prefix!)

Recall from previous classes that complementary angles add up to 90°. For example, 70° and 20° are complementary angles, as are 40° and 50°.

The Complementary Angle Theorem states that cofunctions of complementary angles are equal. So, for example, $\sin(70°) = \cos(20°)$ and $\cot(40°) = \tan(50°)$.

Another way to write this would be: $[\text{trig function of } \theta] = [\text{trig cofunction of } (90° - \theta)]$.

Or in radians, we would have: $[\text{trig function of } \theta] = [\text{trig cofunction of } \left(\frac{\pi}{2} - \theta \right)]$.

Example: Find the exact value of each expression.

a) $\frac{\cos 40°}{\sin 50°}$

b) $\cot 60° - \frac{\sin 30°}{\sin 60°}$

c) $\cot 25° \cdot \csc 65° \cdot \sin 25°$