FIND THE EXACT VALUES OF THE TRIG FUNCTIONS OF 45° ($\pi/4$ radians)

Given a right triangle with one angle of 45°, what is the measurement of the other angle? __________ What does this tell you about the lengths of the legs of the triangle (a and b)? __________________________________________________

So if side $a$ has a length of 1, the length of side $b$ is ________ and, using the Pythagorean Theorem, we find that the length of side $c$ (the hypotenuse) is ________.

This type of triangle, which, considering the angle measurements in degrees is called a _____ - _____ - _____ triangle, is commonly used in both trigonometry and geometry. If we considered the same triangle with its angles measured in radians, it would be called a _____ - _____ - _____ triangle. You need to memorize that the legs of this type of triangle are each equal to ______ and its hypotenuse is equal to ________.

Example: Find the values of the following trig functions. Rationalize denominators where necessary.

a) $\sin\left(\pi/4\right)$  
b) $\cos\left(\pi/4\right)$  
c) $\tan\left(\pi/4\right)$  
d) $\csc\left(\pi/4\right)$  
e) $\sec\left(\pi/4\right)$  
f) $\cot\left(\pi/4\right)$
FIND THE EXACT VALUES OF THE TRIG FUNCTIONS OF $30^\circ \left( \frac{\pi}{6} \text{ radians} \right)$ AND $60^\circ \left( \frac{\pi}{3} \text{ radians} \right)$

Given a $30^\circ$ - $60^\circ$ - $90^\circ$ triangle, with the hypotenuse of length $c = 2$ as shown in the figure to the left, what would be the length of the base $(a)$?  We can't tell using only this triangle. But if we attached its mirror image (as shown in the figure to the right), it would create an equilateral triangle. What do you know about equilateral triangles?

So, what would be the length of the entire base of the equilateral triangle on the right? ________

Then, what would be the length of the base of the triangle in the figure on the left? ________

Now that you know the length of the hypotenuse, $c$, and the base, $a$, use the Pythagorean Theorem to find the length of the remaining leg, $b$.

$b = ________$

So a $30^\circ$ - $60^\circ$ - $90^\circ$ triangle, which in radians would be called a ______ - ______ - ______ triangle, has the following side measurements:

Short Leg = ________

Long Leg = ________

Hypotenuse = ________

You need to memorize the lengths of the sides of this type of triangle!!

Label the lengths of the sides of the triangle to the left. Use this triangle to fill in the table below. Remember "Soh Cah Toa"!

<table>
<thead>
<tr>
<th>$\theta$ (Radians)</th>
<th>$\theta$ (Degrees)</th>
<th>$\sin \theta$</th>
<th>$\cos \theta$</th>
<th>$\tan \theta$</th>
<th>$\csc \theta$</th>
<th>$\sec \theta$</th>
<th>$\cot \theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\pi}{6}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{\pi}{3}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Example: Find the exact value of each expression.

\begin{align*}
\text{a)} & \quad 2 \sin 45^\circ + 4 \cos 30^\circ \\
\text{b)} & \quad \sec \left( \frac{\pi}{4} \right) + 2 \csc \left( \frac{\pi}{3} \right) \\
\text{c)} & \quad \sec^2 \left( \frac{\pi}{3} \right) - \tan^2 \left( \frac{\pi}{4} \right) \\
\text{d)} & \quad 1 + \tan^2 30^\circ - \csc^2 45^\circ
\end{align*}

USING A CALCULATOR TO APPROXIMATE THE VALUES OF THE TRIG FUNCTIONS OF ACUTE ANGLES

You will not be using your calculator very much in this course, but on occasion you will need to be able to find decimal approximations of the trig functions of angles (for instance, angles other than 30°, 45°, and 60°). When using your calculator, you just have to set the MODE to either radians or degrees, depending on the information given in the problem. Also, your calculator has buttons for sine, cosine, and tangent, but NOT for cosecant, secant, and cotangent. So, to enter these reciprocal functions, you have to put them in as follows: \( \csc \theta = 1/\sin \theta \), \( \sec \theta = 1/\cos \theta \), and \( \cot \theta = 1/\tan \theta \).

Example: Use a calculator to find the approximate value of each expression. Round the answer to two decimal places.

\begin{align*}
\text{a)} & \quad \cos 14^\circ \text{ (make sure calculator is in DEGREE MODE)} \\
\text{b)} & \quad \tan 1 \text{ (make sure calculator is in RADIANS MODE)}
\end{align*}
Example (continued):

c) \(\csc 21^\circ\) (in DEGREE MODE, enter as shown in the figure to the right)

d) \(\sec\left(\frac{\pi}{12}\right)\)

MODEL AND SOLVE APPLIED PROBLEMS INVOLVING RIGHT TRIANGLES

Example:
A surveyor can measure the width of a river by setting up a transit at a point \(C\) on one side of the river and taking a sighting of a point \(A\) on the other side. Refer to Figure 37. After turning through an angle of 90° at \(C\), the surveyor walks a distance of 200 meters to point \(B\). Using the transit at \(B\), the angle \(\beta\) is measured and found to be 20°. What is the width of the river rounded to the nearest meter?

Figure 37

Example:
Adorning the top of the Board of Trade building in Chicago is a statue of Ceres, the Roman goddess of wheat. From street level, two observations are taken 400 feet from the center of the building. The angle of elevation to the base of the statue is found to be 55.1° and the angle of elevation to the top of the statue is 56.5°. See Figure 36(a). What is the height of the statue?

Figure 36(a)