Density
The density ($\rho$) of a material is defined as its mass divided by its volume ($\rho = m/v$). We use the lower-case Greek letter rho ($\rho$) to represent the density. The standard SI unit of density is kg/m$^3$. In chemistry and other fields, one typically works with small samples and so it is more convenient to express densities in smaller units. Grams per cubic centimeter, or g/cm$^3$ is a commonly used density unit. A simple conversion calculation will show that the two units are related by a factor of 1000, so that, for example, the density of aluminum is 2710 kg/m$^3$ = 2.71 g/cm$^3$. If you've taken chemistry, you may recall that the density of fresh water is 1000 kg/m$^3$ = 1.000 g/cm$^3$. We will use g/cm$^3$ in this lab.

For this experiment, we measure the dimensions (in cm) and masses (in grams) of several geometrically-shaped samples (cubes, rectangular parallelepipeds and cylinders), and then determine their densities by dividing their masses (in grams) by their calculated volumes (in cm$^3$).

We then compare our experimentally determined values to known true values using the following equation:

$$\% \text{ error} = \left[ \frac{\text{Experimental Value} - \text{Known Value}}{\text{Known Value}} \right] \times 100$$

Note that if your experimental value is larger than the known value, your % error will be positive, whereas if your experimental value is smaller than the known value, you will get a negative % error.

In the case that you do not have a "Known Value" for your specimen, you can compare your experimental value to another experimental value using the following equation:

$$\% \text{ difference} = \left| \frac{\text{Value 1} - \text{Value 2}}{\text{Value 1} + \text{Value 2}} \right| \times 200$$

Note that, in this case, we take the absolute value of the fraction so the % difference is always positive. This is because there is no reference and the fraction in the absolute value bars will change sign depending on which experimental value is (arbitrarily) chosen to be Value 1.

We will be using a vernier caliper to measure lengths. The vernier caliper uses a second scale to provide one or two more digits of precision in a length measurement. Other scientific measurement instruments (like barometers and some astronomical instruments) often incorporate a vernier scale to increase measurement precision. Our vernier caliper will offer us a precision of 10$^{-2}$ cm (hundredths of a centimeter). There are many tutorials available on the www to get practice with reading vernier scales.

To measure the mass of our samples, we will be using a triple-beam balance, which is a standard laboratory instrument. Our balances offer a precision of tenths of grams. It is important that you understand how to handle significant figures in lab calculations. You will be combining many numbers of different precisions and it is important that you not overstate the precision of your calculated result. Please download and read the handout on significant figures available at our class webpage and ask me if you have any questions about them.

It is also important to recognize that density is not a unique identifier. A clear, odorless liquid with density equal to exactly 1.000 g/cm$^3$ is not necessarily water. It might be a different substance, or a mixture of substances whose average density is also 1.000 g/cm$^3$. Mixtures of metals, called alloys, can be designed to have the same density as silver or gold. To precisely identify materials is a job for chemists...
Balancing Vectors
An object at rest with no acceleration is said to be in Static Equilibrium. The vector sum of the forces acting on that object must be zero. In this lab, we apply three different forces at different angles to a small ring which is in equilibrium and analyze the three forces mathematically to verify that they do, indeed, add up to zero, within the limits of experimental accuracy. The forces will be provided by hanging masses. This is basically an exercise in adding vectors.

Consider three forces $A$, $B$ and $C$, such that $A + B + C = 0$. If we first add up $A$ and $B$ and call the sum the resultant $R$ ($A + B = R$), then it must be true that $C = -R$, where the vector $C$ is called the equilibrant of ($A + B$). The equilibrant must have the same magnitude as the resultant $R = (A + B)$ and be directed 180° away from the resultant.

As an example, consider that we hang a 250 gram (0.25 kg) mass at an angle of 15.0° and a mass of 350 grams (0.35 kg) at an angle of 135°. What mass, at what angle, will be required to balance out the first two masses? The weights ($W = mg$) of the two masses are 2.45 N and 3.43 N. So we have

$A = 2.45 \text{ N @ 15.0°}$
$B = 3.43 \text{ N @ 145°}$.

What must $C$ be in order to balance out ($A + B$)?

The technique is to find the $x$ and $y$ components of $A$ and $B$ ($A_x$, $A_y$, $B_x$, $B_y$) and then it must be true that $C_x = -(A_x + B_x)$ and $C_y = -(A_y + B_y)$. We have

$A_x = A \cos \theta_A = 2.45 \text{ N} \cos (15.0°) = 2.37 \text{ N.}$
$A_y = A \sin \theta_A = 2.45 \text{ N} \sin (15.0°) = 0.634 \text{ N}$
$B_x = B \cos \theta_B = 3.43 \text{ N} \cos (145°) = -2.81 \text{ N.}$
$B_y = B \sin \theta_B = 3.43 \text{ N} \sin (145°) = 1.97 \text{ N.}$

Then $C_x = (A_x + B_x) = -0.44 \text{ N}$ and $C_y = (A_y + B_y) = 2.60$ (Note use of sig figs rules)

Finally, we combine $C_x$ and $C_y$ to get

$C = (C_x^2 + C_y^2)^{1/2} = 2.64 \text{ N}$ and $\theta_C = \tan^{-1} (C_y / C_x) = -80.4°$

A weight of 2.64 N corresponds (on Earth) to a mass of 0.269 kg, so we expect that a mass of 269 grams hung at an angle of -80.4° will balance out the first two masses.

In the actual experiment, there will be some error due to the precision of the instrument and the proper use of significant figures. We can quantify this error by comparing the $C_x$ and $C_y$ values of our experimental results to the $C_x$ and $C_y$ values that we calculated above.
**Hooke's Law**

In the Hooke's Law experiment, we progressively add masses to a vertically mounted spring and observe how much the spring stretches in response to the **weight** of the masses. Remember weight is \( W = mg \), so we will have to multiply our masses (in kg) by "\( g \)"; the acceleration due to gravity (9.8 m/s\(^2\)), to get the **force** of gravity stretching the spring (in Newtons). We then plot a graph with the applied force (in N) on the vertical axis and the amount of stretch (in meters) on the horizontal axis of a graph using appropriate graphing software.

Recalling Hooke's Law:

\[
F = kx
\]

where

- \( F \) is the force (in N),
- \( x \) is the amount of stretch or compression (in m),
- \( k \) is the spring constant or stiffness (in N/m)

If Hooke's Law applies to the spring we use, the graphed data will yield a straight line, where the **slope** of the line (in N/m) gives us the spring constant, \( k \), for the spring.

The software we use to plot our data will automatically find the "**Line of Best Fit**" (a.k.a. **Regression Line**), which is the slope and y intercept \( (y = mx + b) \) of the line that best fits the date. The software also calculates for us two very useful numbers which tell us how close the line of best fit approximates our experimental data.

The **Correlation Coefficient** of a best fit line is a **number between 0 and 1** which tells us how well the data fits the line. A correlation coefficient of 1 means the data is perfectly linear and a correlation coefficient of 0 means the data is completely random.

The **RMS Error** tells us what the **average difference** between the experimental data points and the mathematical relationship define by the line of best fit. Values for the RMS error are in the same units as the dependent variable, so they vary depending on the units you're graphing on the y axis.

**The Effect of Air Resistance on the Acceleration of a Free-Falling object**

In this experiment, we investigate the effects of air resistance on the motion of a light ball. The ball will be thrown vertically upward and allowed to fall back down upon a motion sensor. The computer-connected, sonar-based motion sensor will detect the ball's motion and provide us with graphs of the ball's position, velocity and acceleration.

In the absence of air resistance, we expect the ball to have a constant downward acceleration of 9.8 m/s\(^2\) (32 ft/s\(^2\)), whether it is rising, at the apex, or falling. So the velocity will decrease by 9.8 m/s every second on the way up and increase 9.8 m/s\(^2\) on the way down. With the introduction of the air resistance force (viscous friction), we expect slightly different results. Recall that the air resistance force is always **opposite the direction of motion**. On the way up, gravity pulls the ball downward and the air resistance force is downward, so the two forces add up and the real acceleration can be expected to be larger in magnitude than 9.8 m/s\(^2\). Conversely, on the way down, the gravitational force is still downward, but the air resistance force is upward. Here, the two forces are in opposite directions, so they subtract, and the expected acceleration of the ball will be less than 9.8 m/s\(^2\) in magnitude.

The three graphs of the motion (position, velocity, acceleration) will appear very different. The position curve will look like a parabola, reflecting the equation of motion \( \Delta y = v_{yi}t + \frac{1}{2}at^2 \). The coefficient of the quadratic term of this equation should be slightly larger than 4.8 m/s\(^2\) in magnitude on the way up and slightly smaller than 4.8 m/s\(^2\) in magnitude on the way down. The velocity curve will look like a straight line, reflecting the equation \( v_{yt} = v_{yi} + at \), where the slope of the line will be negative and slightly larger than 9.8 m/s\(^2\) in magnitude on the way up and slightly smaller than 9.8 m/s\(^2\) in magnitude on the way down. The acceleration curve will look almost like a horizontal line, with the y value of the line negative and slightly larger than 9.8 m/s\(^2\) in magnitude on the way up and slightly smaller than 9.8 m/s\(^2\) in magnitude on the way down.